Bias Correction for Nonlinear Panel: Why, When and How

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Outline

- Fixed effects in nonlinear panel
  - Incidental parameter problem
  - A consistency or a bias issue?
- Key existing results and algorithms
  - Where does the bias come from?
  - How to correct the bias?
    1. Analytical bias correction
    2. Jackknife bias correction
- Work in progress: crossover jackknife
  - Why? Dealing with nonstationary regressors
  - Calibrated simulations:
    1. Causal impact of democracy on output growth
    2. Labor force participation (appendix)

Motivation

- Fixed effects are widely used in applied research

$$y_{i,t} = x_{i,t}\beta + \alpha_i + \varepsilon_{i,t}$$

- Assuming strict exogeneity $\mathbb{E}(\varepsilon_{i,t} \mid x_{i,1}, \ldots, x_{i,T}, \alpha_i) = 0$, unbiased, consistent and asymptotically normal in static linear settings
- Not so in nonlinear panel, e.g.
  1. Dynamic linear

$$y_{i,t} = \theta y_{i,t-1} + \alpha_i + \varepsilon_{i,t}, \quad \varepsilon_{i,t} \mid (y_{i,1}, \ldots, y_{i,t-1}), \alpha \sim \mathcal{N}(0, \sigma^2)$$

  2. Panel binary response single index

$$y_{i,t} = \mathbb{1}(\theta x_{i,t} + \alpha_i \geq \varepsilon_{i,t}), \quad \varepsilon_{i,t} \mid (x_{i,1}, \ldots, x_{i,t}), \alpha \sim F_\varepsilon$$

  where F_ε is a known CDF like standard normal or logistic
- Why? Incidental parameter problem

Neyman and Scott (1948)

Consider independent $\{X_{i,t}\}$, $i = 1, \ldots, N$ and $t = 1, \ldots, T$ where

$$X_{i,t} \sim \mathcal{N}(\mu_i, \sigma_0^2)$$

Short panel: T is fixed and $N \longrightarrow \infty$

Goal: estimate σ_0^2 consistenty

- MLE estimators:

$$\hat{\mu}_i = \overline{X}_i := \frac{1}{T}\sum_{t=1}^{T} X_{i,t}, \quad \hat{\sigma}^2 = \frac{1}{NT}\sum_{i=1}^{N}\sum_{t=1}^{T}(X_{i,t} - \overline{X}_i)^2$$

- By LLN, $\hat{\sigma}^2 \xrightarrow{p} \sigma_0^2 \frac{T-1}{T}$, **inconsistent**
- Why? μ_i is nuisance whose dimension grows with N (incidental parameters). With T fixed, the estimate is noisy and affects $\hat{\sigma}^2$
- Can adjust $\hat{\sigma}^2$ by multipling $\frac{T}{T-1}$, in general hard

How Does it Relate to Panel? Nickell Bias (1981)

$$y_{i,t} = \theta y_{i,t-1} + \alpha_i + \varepsilon_{i,t}, \quad |\theta| < 1; \varepsilon_{i,t} \overset{iid}{\sim} \mathcal{N}(0, \sigma^2); i = 1, \ldots, N; t = 1, \ldots, T$$

- Fixed effect estimator:

$$\widehat{\theta}_{FE} = \sum_{t=1}^{T} \sum_{i=1}^{N} (y_{i,t-1} - \overline{y}_{i-1})(y_{i,t} - \overline{y}_i) \Big/ \sum_{t=1}^{T} \sum_{i=1}^{N} (y_{i,t-1} - \overline{y}_{i-1})^2$$

- Fix T and set $N \longrightarrow \infty$,

$$\widehat{\theta}_{FE} - \theta \overset{p}{\longrightarrow} -\frac{1+\theta}{T-1}\left(1 - \frac{1}{T}\frac{1-\theta^T}{1-\theta}\right)\left[1 - \frac{2\theta}{(1-\theta)(T-1)}\left(1 - \frac{1}{T}\frac{1-\theta^T}{1-\theta}\right)\right]^{-1}$$

- If $\theta > 0$, $\widehat{\theta}_{FE}$ is biased downward
  - $-\frac{1+\theta}{2}$ when $T = 2$ and approximately $-\frac{1+\theta}{T-1}$ for a large T
- Why: $\overline{y}_{i-1}$ is correlated with $\overline{\varepsilon}_i$ because the latter contains $\varepsilon_{i,t-1}$

Some Key Questions and Partial Answers

- Inconsistent or biased?
  - If T is fixed, inconsistent: $N \longrightarrow \infty$ doesn't alleviate the problem
  - If $T \longrightarrow \infty$ as well, becomes a bias problem (large T asymptotics)
- How to deal with the bias?
  - Analytical bias correction
  - Jackknife bias correction
  - Indirect inference
- Does bias correction affect the variance? Yes and no
- Time fixed effect? Another incidental parameter problem
- How to deal with nonstationary regressors? Crossover jackknife

Fixed Effect Model

- Data: $z_{i,t} = (y_{i,t}, x_{i,t})$, $i = 1, \dots, N; t = 1, \dots, T$
- $x_{i,t}$ can be either strictly exogenous or predetermined w.r.t. $y_{i,t}$
- Known density function of $z_{i,t}$:

$$f(z_{i,t}; \theta_0, \alpha_{i0})$$

  - θ_0: finite–dimensional parameter of interest
  - α: unobserved individual fixed effects

- Fixed effect estimation:

$$(\theta, \alpha_1, \dots, \alpha_N) = \arg\max_{\theta, \alpha_i} \frac{1}{NT} \sum_{i=1}^{N} \sum_{t=1}^{T} \ln f(z_{i,t}; \theta, \alpha_i)$$

- Reminder: in static linear panel demeaning removes α_i's

Fixed Effect Estimation

- **FE estimator**:

$$\widehat{\theta}_{FE} = \arg\max_{\theta \in \Theta} \frac{1}{NT} \sum_{i=1}^{N} \sum_{t=1}^{T} \ln f(z_{i,t}; \theta, \widehat{\alpha}_i(\theta)),$$

where

$$\widehat{\alpha}_i(\theta) = \arg\max_{\alpha \in \mathcal{A}} \frac{1}{T} \sum_{t=1}^{T} \ln f(z_{i,t}; \theta, \alpha_i)$$

Fixed Effect Estimation

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$$\widehat{\theta}_{FE} = \arg\max_{\theta \in \Theta} \frac{1}{NT} \sum_{i=1}^{N} \sum_{t=1}^{T} \ln f(z_{i,t}; \theta, \widehat{\alpha}_i(\theta)),$$

where

$$\widehat{\alpha}_i(\theta) = \arg\max_{\alpha \in \mathcal{A}} \frac{1}{T} \sum_{t=1}^{T} \ln f(z_{i,t}; \theta, \alpha_i)$$

- Define $\overline{\mathbb{E}}(m(z_{i,t}, \alpha_i)) := \lim_{N \to \infty} \frac{1}{N} \sum_{i=1}^{N} \mathbb{E}(m(z_{i,t}, \alpha_i))$, then

$$\widehat{\theta}_{FE} \xrightarrow{p} \theta_T := \arg\max_{\theta \in \Theta} \overline{\mathbb{E}}\left(\sum_{t=1}^{T} \ln f(z_{i,t}; \theta, \widehat{\alpha}_i(\theta)) \right)$$

Fixed Effect Estimation

- **FE estimator**:

$$\widehat{\theta}_{FE} = \arg\max_{\theta\in\Theta} \frac{1}{NT} \sum_{i=1}^{N} \sum_{t=1}^{T} \ln f(z_{i,t}; \theta, \widehat{\alpha}_i(\theta)),$$

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$$\widehat{\theta}_{FE} \xrightarrow{p} \theta_T := \arg\max_{\theta\in\Theta} \overline{\mathbb{E}}\left(\sum_{t=1}^{T} \ln f(z_{i,t}; \theta, \widehat{\alpha}_i(\theta)) \right)$$

- Incidental parameter problem:

$$\theta_T \neq \theta_0 := \arg\max_{\theta\in\Theta} \overline{\mathbb{E}}\left(\sum_{t=1}^{T} \ln f(z_{i,t}; \theta, \alpha_{i0}) \right)$$

- Under smooth likelihood: $\theta_T = \theta_0 + \frac{B}{T} + \mathcal{O}(\frac{1}{T^2})$

Large T Asymptotic Distribution

Assume $N, T \longrightarrow \infty, N/T \longrightarrow \kappa$, under regularity conditions,

$$\sqrt{NT}(\widehat{\theta}_{FE} - \theta_0) \xrightarrow{d} \mathcal{N}(B\sqrt{\kappa}, V)$$

Large T Asymptotic Distribution

Assume $N, T \longrightarrow \infty, N/T \longrightarrow \kappa$, under regularity conditions,

$$\sqrt{NT}(\widehat{\theta}_{FE} - \theta_0) \xrightarrow{d} \mathcal{N}(B\sqrt{\kappa}, V)$$

Interpretations

- B: due to using $\widehat{\alpha}(\theta)$; DGP dependent
- If T and N increase at the same rate, bias persists asymptotically

Large T Asymptotic Distribution

Assume $N, T \longrightarrow \infty, N/T \longrightarrow \kappa$, under regularity conditions,

$$\sqrt{NT}(\widehat{\theta}_{FE} - \theta_0) \xrightarrow{d} \mathcal{N}(B\sqrt{\kappa}, V)$$

Interpretations

- B: due to using $\widehat{\alpha}(\theta)$; DGP dependent
- If T and N increase at the same rate, bias persists asymptotically

Remedies

- Recentering: estimate B and construct $\tilde{\theta} = \widehat{\theta}_{FE} - \widehat{B}/T$
- Analytically: model–specific derivation
- Nonparametrically: panel jackknife

ABC for Neyman and Scott

- Recall that

$$\sigma_T^2 := \operatorname*{plim}_{N\to\infty} \widehat{\sigma}^2 = \sigma_0^2 \frac{T-1}{T} = \sigma_0^2 - \frac{\sigma_0^2}{T}$$

- Rewrite to be:

$$\sigma_0^2 = \sigma_T^2 + \frac{\sigma_0^2}{T}$$

- Bias term: $B = -\sigma_0^2$
- Analytical bias correction estimator:

$$\tilde{\sigma}^2 = \widehat{\sigma}^2 + \frac{\widehat{\sigma}^2}{T}$$

- Leading bias removed:

$$\operatorname*{plim}_{N\to\infty} \tilde{\sigma}^2 = \sigma_0^2 - \frac{\sigma_0^2}{T^2}$$

▸ ABCs for some models

Delete–One Panel Jackknife (Hahn and Newey, 2004)

- Recall $\theta_T = \theta_0 + \frac{B}{T} + \mathcal{O}(\frac{1}{T^2})$ and want an estimate $\widehat{B}/T$
- Denote $\widehat{\theta}_{(t)}$ as the fixed effects estimator using subsample without time t observations
- Jackknife estimator:

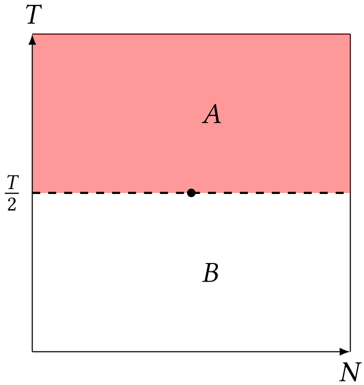
$$\tilde{\theta} = \widehat{\theta}_{FE} - (T-1)\left(\frac{1}{T}\sum_{t=1}^{T}\widehat{\theta}_{(t)} - \widehat{\theta}_{FE}\right) = T\widehat{\theta}_{FE} - \frac{T-1}{T}\sum_{t=1}^{T}\widehat{\theta}_{(t)}$$

- $\mathcal{O}(T^{-1})$ bias correction:

$$\operatorname*{plim}_{N\to\infty} \tilde{\theta} = T\theta_T - (T-1)\theta_{T-1} = \theta_0 + \mathcal{O}\left(\frac{1}{T^2}\right)$$

- Under regularity conditions, $\sqrt{NT}(\tilde{\theta} - \theta_0) \xrightarrow{d} \mathcal{N}(0, V)$
- Caveat: can't allow for dependencies among obs of the same unit
- Remedy? Block subpanels

Dhaene and Jochmans (2015): Split–Panel Jackknife



- Split–panel bias correction estimator:

$$\tilde{\theta} = 2\widehat{\theta}_{FE} - \frac{1}{2}(\widehat{\theta}_A + \widehat{\theta}_B)$$

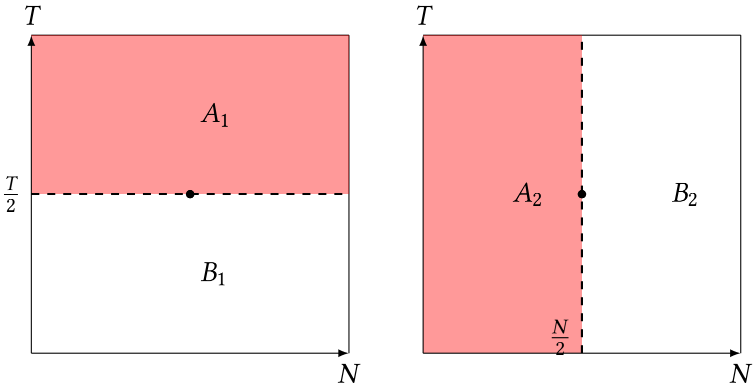
- Theorem 3.1: $\lim_{N\to\infty} \tilde{\theta} = \theta_0 + o(T^{-1})$, as $N, T \longrightarrow \infty$ and $N/T \longrightarrow \kappa$,

$$\sqrt{NT}(\tilde{\theta} - \theta_0) \xrightarrow{d} \mathcal{N}(0, V)$$

Split–Panel Jackknife: Some Remarks

- No variance inflation due to averaging
- Leading bias removed, absolute higher–order biases ↑ ▸ Intuition
  - Such increases are minimized for **almost–equal** partition
- Limitation: imposes stationarity along the time series, rules out nonstationary initial observations, time fixed effects, nonstationary regressors

Fernández-Val and Weidner (2016): Adding Time FE



- Density function is now $f(z_{i,t}; \theta_0, \alpha_{i0}, \gamma_{t0})$
- An additional bias term: $\theta_T = \theta_0 + \frac{B}{T} + \frac{D}{N} + \mathcal{O}(T^{-1} \vee N^{-1})$
- Split–panel bias correction (SBC) estimator ▸ Intuition

$$\tilde{\theta}_{SBC} = 3\widehat{\theta}_{FE} - \frac{1}{2}(\widehat{\theta}_{A_1} + \widehat{\theta}_{B_1}) - \frac{1}{2}(\widehat{\theta}_{A_2} + \widehat{\theta}_{B_2})$$

- Stata command: **probitfe** and **logitfe**

Unconditional Homogeneity

Key assumption (Unconditional homogeneity): The sequence

$$\{(z_{i,t}, \alpha_i, \gamma_t) \,:\, 1 \leq i \leq N, 1 \leq t \leq T\}$$

is identically distributed across i and strictly stationary across t for each N, T

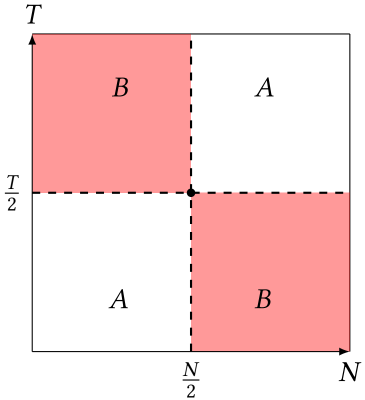
- Initial condition needn't come from stationary distribution of $y_{i,t}$
- Rules out trends in unobserved effects or observed variables

Allow Nonstationary Regressors: Crossover

Why should we care? Empirically relevant

- Age and income, two commonly used regressors, are trended
- Some policy variables (e.g., closing gyms in April) are nonstationary: first half of panel is zero, the other half takes one

Graphical Illustration of Crossover Splitting



- Each subpanel uses all cross-section units
- Crossover bias correction (CBC) estimator:

$$\tilde{\theta}_{CBC} = 2\widehat{\theta}_{FE} - \frac{1}{2}(\widehat{\theta}_A + \widehat{\theta}_B)$$

Calibration Exercise I: Causal Effect of Democracy

Data: Acemoglu et al. (2019)

- Balanced panel of 147 countries from 1987–2009

Regression Specification:

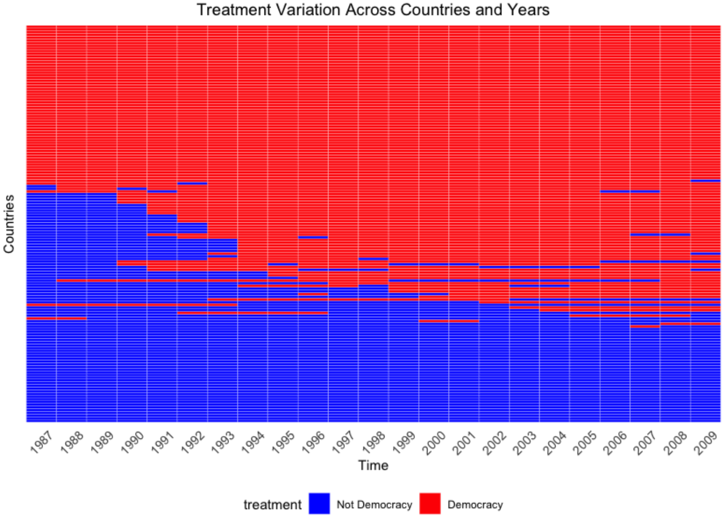
$$y_{i,t} = \alpha_i + \gamma_t + D'_{i,t}\theta + W'_{i,t}\beta + \varepsilon_{i,t}, \quad i = 1, \ldots, N; \quad t = 1, \ldots, T$$

- $y_{i,t}$: log GDP for a country i in year t
- $D_{i,t}$: democracy binary index
- $W_{i,t}$: controls including a constant and **four lags of $y_{i,t}$**
- α_i and γ_t: unobserved unit and time effects
- $\varepsilon_{i,t}$: error term normalized to have zero mean for each unit that satisfies the **weak sequential exogeneity** condition

$$\varepsilon_{i,t} \perp \mathcal{I}_{i,t}, \quad \mathcal{I}_{it} := \left\{ (D_{i,s}, W_{i,s}, \gamma_s)_{s=1}^{t}, \alpha_i \right\}$$

- $Z_i := \left\{ (y_{i,t}, D'_{i,t}, W'_{i,t})' \right\}_{t=1}^{T}$ i.i.d. across i

Nonstationarity of Democracy Index



- Acknowledgement: graph generated using R package PanelMatch
- Package for Imai, Kim and Wang (2020)

Calibration Procedure

1. Run the regression using real panel data
2. Simulate $\{y_{i,t}\}$ using coefficients in step one
3. Construct simulated panel data using synthetic $\{y_{i,t}\}$ and real covariates
4. Use simulated panel data to run the regression Compute uncorrected FE, ABC, SBC and CBC. Compute bootstrap standard errors ▸ Setting Seed
5. Repeat steps two–four 500 times
6. Report diagnostic statistics:
   - Bias, standard deviation, RMSE, BSE/SD, p-value coverage, length
   - *SD*: standard deviation of simulated estimator

R code is available here.

Calibrated Dynamic Linear, N = 147, T = 19

	Bias	Std Dev	RMSE	BSE/SD	p.95 (BSE)	Length (BSE)
			Coefficient of dem			
FE	-3.02	22.99	23.17	1.00	0.94	0.91
SBC	-6.11	29.95	30.54	0.99	0.96	1.16
ABC	-1.96	23.39	23.45	1.00	0.95	0.92
CBC	0.87	29.73	29.71	0.96	0.93	1.12
			Coefficient of l1lgdp			
FE	-2.10	1.31	2.47	1.01	0.62	0.05
SBC	5.28	1.54	5.50	1.03	0.08	0.06
ABC	-0.39	1.43	1.49	1.00	0.94	0.06
CBC	0.09	1.40	1.40	1.01	0.96	0.06
			Coefficient of l2lgdp			
FE	-17.00	22.30	28.02	0.97	0.84	0.84
SBC	-8.25	24.04	25.39	0.98	0.92	0.92
ABC	-5.24	23.28	23.84	0.96	0.94	0.87
CBC	-0.86	23.47	23.46	0.96	0.95	0.88
			Coefficient of l3lgdp			
FE	-8.59	36.71	37.67	0.96	0.94	1.39
SBC	-7.62	39.78	40.47	0.98	0.95	1.53
ABC	-2.63	37.88	37.93	0.96	0.94	1.43
CBC	1.21	38.28	38.27	0.97	0.93	1.45
			Coefficient of l4lgdp			
FE	7.58	16.28	17.94	0.98	0.92	0.62
SBC	11.92	18.04	21.61	0.99	0.88	0.70
ABC	5.80	16.63	17.60	0.98	0.93	0.64
CBC	1.50	18.13	18.18	0.98	0.92	0.70
			Coefficient of long run effect			
FE	-6.48	22.32	23.22	1.01	0.94	0.88
SBC	19.03	31.22	36.54	0.99	0.88	1.22
ABC	-3.07	23.43	23.61	1.0	0.96	0.92
CBC	0.15	29.17	29.14	0.97	0.93	1.10

Conclusions

- Fixed effects estimation in nonlinear panel suffers from incidental parameter problem
- In long panel, becomes a bias issue and the leading bias can be corrected analytically or by jackknife
- To accommodate nonstationarity in the observed covariates, we propose a crossover jackknife method

ABC for Simple Models

AR(1) Panel (Hahn and Kuersteiner, 2002):

$$y_{i,t} = \alpha_i + \theta_0 y_{i,t-1} + \varepsilon_{i,t}, \quad \varepsilon_{i,t} \overset{iid}{\sim} \mathcal{N}(0, \Omega)$$

$$|\theta_0| < 1, \quad \frac{1}{n}\sum_{i=1}^{n} y_{i,0}^2 = O(1), \quad \frac{1}{n}\sum_{i=1}^{n} \alpha_i^2 = O(1)$$

- ABC: $\tilde{\theta}_{ABC} = \widehat{\theta}_{FE} + \frac{1}{T}(1 + \widehat{\theta}_{FE})$
- Doesn't work well with unit root (next slide)

Probit (Fernández-Val, 2009):

$$y_{i,t} = \mathbb{1}\{X_{i,t}'\theta_0 + \alpha_i > \varepsilon_{i,t}\}, \quad \varepsilon_{i,t} \overset{iid}{\sim} \mathcal{N}(0, 1)$$

- $X_{i,t}$: strictly exogeneous single regressor
- ABC: $\tilde{\theta}_{ABC} = \widehat{\theta}_{FE} - \frac{1}{T}\frac{\widehat{\sigma}^2 \widehat{\theta}_{FE}}{2}$, where
  $\widehat{\sigma}^2 = \frac{1}{T}\sum_{t=1}^{T}\left(\phi^2(X_{i,t}'\widehat{\theta}_{FE} + \widehat{\alpha}_i)/[\Phi(X_{i,t}'\widehat{\theta}_{FE} + \widehat{\alpha}_i)(1 - \Phi(X_{i,t}'\widehat{\theta}_{FE} + \widehat{\alpha}_i))]\right)^{-1}$

◂ Back

Hahn and Kuersteiner (2002) Unit Root

When $|\theta_0| < 1$, asymptotic distribution of $\tilde{\theta}_{ABC}$ is

$$\sqrt{NT}(\tilde{\theta}_{ABC} - \theta_0) \xrightarrow{d} \mathcal{N}(0, 1 - \theta_0^2)$$

Now suppose $\varepsilon_{i,t} \overset{iid}{\sim} \mathcal{N}(0, \sigma^2)$, $\lim \frac{1}{N} \sum_{i=1}^N \alpha_i^2 > 0$, $\theta_0 = 1$, $\lim \sqrt{N/T} = \kappa$

$$\sqrt{NT^3}(\widehat{\theta} - \theta_0) \xrightarrow{d} \mathcal{N}\left(-\frac{6\sigma^2\kappa}{\lim \frac{1}{N} \sum_{i=1}^N \alpha_i^2}, \frac{12\sigma^2}{\lim \frac{1}{N} \sum_{i=1}^N \alpha_i^2} \right)$$

Asymptotic bias depends on $\lim \frac{1}{N} \sum_{i=1}^N \alpha_i^2$

Jackknife in Cross Section

- Delete–1 jackknife sample

$$\boldsymbol{X}_{[i]} = \{X_1, X_2, \ldots, X_{i-1}, X_{i+1}, \ldots, X_n\}$$

- Jackknife replicate: $\widehat{\theta}_{(i)} = g(\boldsymbol{X}_{[i]})$
- Empirical average of jackknife replicates: $\widehat{\theta}_{(\cdot)} = \frac{1}{n}\sum_{i=1}^{n} \widehat{\theta}_{(i)}$
- Jackknife estimate of bias:

$$\widehat{bias}(\widehat{\theta}) := (n-1)(\widehat{\theta}_{(\cdot)} - \widehat{\theta})$$

- Jackknife bias corrected estimate:

$$\widehat{\theta}_{JBC} = \widehat{\theta} - \widehat{bias}(\widehat{\theta}) = n\widehat{\theta} - (n-1)\widehat{\theta}_{(\cdot)}$$

Jackknife Removes Leading Bias

Suppose $\mathbb{E}(\widehat{\theta}) = \theta + \overbrace{\frac{b_1(\theta)}{n} + \frac{b_2(\theta)}{n^2} + \frac{b_3(\theta)}{n^3} + \ldots}^{bias}$, then

$$\mathbb{E}(\widehat{\theta}_{(\cdot)}) = \theta + \frac{b_1(\theta)}{n-1} + \frac{b_2(\theta)}{(n-1)^2} + \frac{b_3(\theta)}{(n-1)^3} + \ldots,$$

leading to $\mathbb{E}(\widehat{bias}(\widehat{\theta})) = \frac{b_1(\theta)}{n} + \frac{(2n-1)b_2(\theta)}{n^2(n-1)} + \frac{(3n^2-3n+1)b_3(\theta)}{n^3(n-1)} + \ldots$, and hence

$$\mathbb{E}(\widehat{\theta}_{JBC}) = \theta - \frac{b_2(\theta)}{n(n-1)} - \frac{(2n-1)b_3(\theta)}{n^2(n-1)^2} + \cdots$$

Jackknife removes the leading bias, but changes the higher order biases

◂ Back

Sketch of SBC Correction

$$\tilde{\theta}_{SBC} - \theta_0 = (\widehat{\theta}_{FE} - \theta_{NT} + \theta_{NT} - \theta_0) - (\tilde{\theta}_{N,T/2} - \widehat{\theta}_{FE}) - (\tilde{\theta}_{N/2,T} - \widehat{\theta}_{FE})$$

- As $N, T \longrightarrow \infty$, $\sqrt{NT}(\widehat{\theta}_{FE} - \theta_{NT}) \xrightarrow{d} \mathcal{N}(0, V)$
- As $N, T \longrightarrow \infty$, for $N/T \longrightarrow \kappa$, $\sqrt{NT}(\widehat{\theta}_{FE} - \theta_0) \xrightarrow{d} \mathcal{N}(B\sqrt{\kappa} + D\sqrt{\kappa}^{-1}, V)$
- Denote $a \vee b := \max\{a, b\}$, for smooth likelihood,

$$\theta_{NT} = \theta_0 + B/T + D/N + \mathcal{O}(T^{-1} \vee N^{-1})$$

- $\tilde{\theta}_{N,T/2} - \widehat{\theta}_{FE} = B/T + o(T^{-1} \vee N^{-1})$
- $\tilde{\theta}_{N/2,T} - \widehat{\theta}_{FE} = D/N + o(T^{-1} \vee N^{-1})$

◂ Back

Calibration Exercise II: Labor Force Participation

Data: Panel Study of Income Dynamics (PSID)

- Women aged 18–60 in 1985 who (1) were continuously married with husbands in the labor force in each of the sample periods; (2) changed LFP status during the sample period 1979–1988
- $N = 664$, $T = 9$

Regression Specification:

$$y_{i,t} = \mathbb{1}\left\{ \theta y_{i,t-1} + \beta X_{i,t} + \alpha_i + \gamma_t - \varepsilon_{i,t} > 0 \right\}$$

- $y_{i,t}$: labor force participation (LFP) indicator
- $y_{i,t-1}$: lagged LFP
- $X_{i,t}$: numbers of children aged 0–2, 3–5 and 6–17; log of husband earnings; age and age squared
- α_i: individual fixed effects
- γ_t: time fixed effects
- $\varepsilon_{i,t} \overset{iid}{\sim} \mathcal{N}(0, 1)$

Calibration Procedure

1. Run the Probit regression using real panel data
2. Simulate $\{y_{i,t}\}$ using coefficients in step one
3. Construct simulated panel data using synthetic $\{y_{i,t}\}$ and real covariates.
4. Use simulated panel data to run Probit regression. Compute uncorrected FE, ABC, SBC and CBC. Compute bootstrap standard errors
5. Repeat steps two–four 500 times
6. Report diagnostic statistics

Calibrated Dynamic Probit, N = 664, T = 9

	Bias	Std Dev	RMSE	BSE/SD	p.95 (BSE)	Length (BSE)
			Coefficient of lag-lfp			
FE	-54.08	6.75	54.50	0.99	0.00	0.26
SBC	-5.74	8.50	10.25	0.99	0.88	0.33
ABC	-10.78	6.68	12.68	0.99	0.63	0.26
CBC	-6.37	8.57	10.67	0.97	0.88	0.32
			Coefficient of kids02			
FE	33.91	14.67	36.94	1.02	0.36	0.58
SBC	6.13	18.55	19.52	1.02	0.93	0.74
ABC	5.99	12.13	13.52	1.02	0.92	0.49
CBC	4.13	17.61	18.07	1.00	0.93	0.69
			Coefficient of kids35			
FE	47.78	25.76	54.27	1.02	0.59	1.03
SBC	20.06	34.63	39.99	1.01	0.91	1.37
ABC	10.16	21.30	23.58	1.02	0.92	0.85
CBC	17.21	33.22	37.39	1.02	0.92	1.33
			Coefficient of kids617			
FE	51.97	86.45	100.80	0.95	0.89	3.22
SBC	28.83	110.29	113.89	1.01	0.95	4.35
ABC	12.76	71.92	72.97	0.95	0.93	2.67
CBC	21.88	108.87	110.94	1.00	0.94	4.27
			Coefficient of log husband income			
FE	27.01	29.78	40.18	1.02	0.86	1.19
SBC	5.02	33.74	34.08	1.01	0.95	1.34
ABC	6.00	25.77	26.43	1.01	0.94	1.02
CBC	5.69	33.58	34.03	0.99	0.95	1.31
			Coefficient of age			
FE	29.49	36.79	47.12	1.03	0.90	1.48
SBC	-3.73	43.73	43.84	1.03	0.96	1.77
ABC	3.72	31.57	31.75	1.01	0.96	1.25
CBC	-1.13	43.38	43.35	1.02	0.96	1.73
			Coefficient of age2			
FE	33.81	29.46	44.82	1.05	0.82	1.21
SBC	-0.17	39.78	39.74	1.03	0.95	1.61
ABC	6.42	24.05	24.87	1.04	0.96	0.98
CBC	1.40	39.44	39.42	1.04	0.96	1.60

Setting Seed within a Function in R

- Set seed in the global environment for panel data simulation
- Set a new seed for bootstrap, otherwise number of bootstraps affects panel data simulation

```
set.seed(88) # seed in global envrionment for panel simulation
# A function to compute bootstrap standard errors
bse <- function(data, form.fe, ncores, btimes, bseed) {
  # store seeds set in the global environment
  old <- .Random.seed
  # upon exiting the function environment,
  # restore to the original seed
  on.exit({.Random.seed <<- old})
  # within this function environment, set
  # the new seed for bootstrap standard errors
  set.seed(bseed, kind = "L'Ecuyer-CMRG")
  result <- boot(data = data, statistic = estimators, sim = "parametric",
               ran.gen = data.rg, mle = 0, form = form.fe,
               parallel = "multicore", ncpus = ncores, R = btimes)
```

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