Abstract

This note presents a counterexample of incomplete information games in which the set of distributions cannot be generated by a 2−alternating capacity. Hence, the robust test framework does not accommodate Bayesian Nash Equilibrium.

Consider the setting of Aguirregabiria and Mira (2019) (henceforth AM19) without the covariates (x, w). Based on their characterization of model predictions in Eq.(5), the set of distributions compatible with a given structure (i.e. payoff, distribution of the unobservable, and BNE) is

$$\mathcal{P} = \left\{ Q : Q(a) = \sum_{\tau \in \Gamma(\pi)} \lambda(\tau) \prod_{i=1}^{N} P_i^{(\tau)}(a),\ P^{(\tau)} = \Psi(P^{(\tau)}) \right\}. \tag{1}$$

The following is a counterexample in which $\mathcal{P}$ is not generated by a 2-alternating capacity.

Example 1. Consider Example 2 (coordination game) in AM19. Suppose $N = 2$, $a_i \in \{0, 1\}$ and the three equilibria are characterized by the following conditional choice probabilities $P_i^{(\tau)}$ for $\tau = 1, 2, 3$:

$$P_i^{(\tau)}(1) = \begin{cases} 0.9 & \tau = 1 \\ 0.7 & \tau = 2 \\ 0.1 & \tau = 3. \end{cases} \tag{2}$$

Compared to Figure 2 in AM19, this situation roughly corresponds to a setting where the teacher's effort is slightly below 0.6.

According to Lemma 2.2 in Bednarski (1982), the 2-alternating property of the upper probability of $\mathcal{P}$ is equivalent to the following: For any monotone sequence of events $A_1 \subset A_2 \subset \cdots \subset A_n$, there exists $P_0 \in \mathcal{P}$ such that for every ℓ,

$$P_0(A_\ell) = \sup_{P \in \mathcal{P}} P(A_\ell).$$

Now take the following sequence

$$\begin{aligned} A_1 &= \{(1, 0)\} \\ A_2 &= \{(1, 0), (0, 1)\} \\ A_3 &= \{(1, 0), (0, 1), (1, 1)\}. \end{aligned}$$

Note that, by simple calculation, for $\ell = 1, 2$,

$$\sup_{P \in \mathcal{P}} P(A_1) = \sum_{a \in A_1} \prod_{i=1}^{N} P_i^{(2)}(a) = 0.21$$
$$\sup_{P \in \mathcal{P}} P(A_2) = \sum_{a \in A_2} \prod_{i=1}^{N} P_i^{(2)}(a) = 0.42.$$

That is, for these events, the maximizing measure is the one induced by selecting the middle equilibrium ($\tau = 2$) with probability 1. However,

$$\sup_{P \in \mathcal{P}} P(A_3) = \sum_{a \in A_3} \prod_{i=1}^{N} P_i^{(1)}(a) = .99 > .91 = \sum_{a \in A_3} \prod_{i=1}^{N} P_i^{(2)}(a).$$

If $\mathcal{P}$ is generated by a 2-alternating capacity, the upper envelope of $P(A_3)$ should still be given by the same equilibrium (i.e. $\tau = 2$), but this is not the case. Hence, $\mathcal{P}$ is not generated by a 2-alternating capacity.

References

Aguirregabiria, V. and P. Mira (2019): "Identification of Games of Incomplete Information with Multiple Equilibria and Unobserved Heterogeneity," *Quantitative Economics*, 10, 1659–1701.

Bednarski, T. (1982): "Binary Experiments, Minimax Tests and 2-Alternating Capacities," *The Annals of Statistics*, 10, 226–232.